

The Regula Falsi Method

Another popular algorithm is the *method of false position* or the *regula falsi method*. It was developed because the bisection method converges at a fairly slow speed. As before, we assume that $f(a)$ and $f(b)$ have opposite signs. The bisection method used the midpoint of the interval $[a, b]$ as the next iterate. A better approximation is obtained if we find the point $(c, 0)$ where the secant line L joining the points $(a, f(a))$ and $(b, f(b))$ crosses the x -axis (see Figure 2.8). To find the value c , we write down two versions of the slope m of the line L :

$$(16) \quad m = \frac{f(b) - f(a)}{b - a},$$

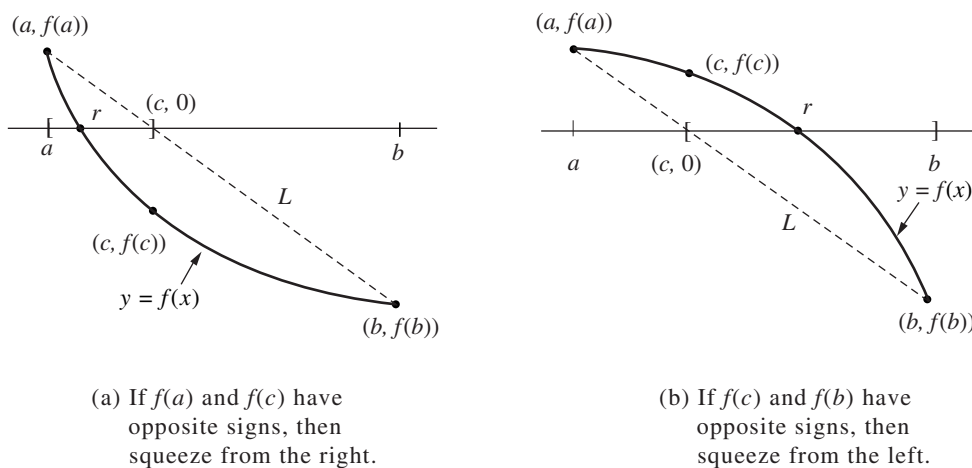


Figure 2.8 The decision process for the false position method.

where the points $(a, f(a))$ and $(b, f(b))$ are used, and

$$(17) \quad m = \frac{0 - f(b)}{c - b},$$

where the points $(c, 0)$ and $(b, f(b))$ are used.

Equating the slopes in (16) and (17), we have

$$\frac{f(b) - f(a)}{b - a} = \frac{0 - f(b)}{c - b},$$

which is easily solved for c to get

$$(18) \quad c = b - \frac{f(b)(b - a)}{f(b) - f(a)}.$$

The three possibilities are the same as before:

(19) If $f(a)$ and $f(c)$ have opposite signs, a zero lies in $[a, c]$.

(20) If $f(c)$ and $f(b)$ have opposite signs, a zero lies in $[c, b]$.

(21) If $f(c) = 0$, then the zero is c .

Convergence of the False Position Method

The decision process implied by (19) and (20) along with (18) is used to construct a sequence of intervals $\{[a_n, b_n]\}$ each of which brackets the zero. At each step the approximation of the zero r is

$$(22) \quad c_n = b_n - \frac{f(b_n)(b_n - a_n)}{f(b_n) - f(a_n)},$$

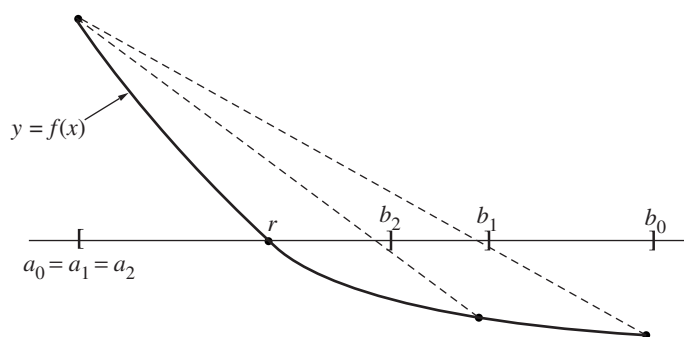


Figure 2.9 The stationary endpoint for the false position method.

and it can be proved that the sequence $\{c_n\}$ will converge to r . But beware; although the interval width $b_n - a_n$ is getting smaller, it is possible that it may not go to zero. If the graph of $y = f(x)$ is concave near $(r, 0)$, one of the endpoints becomes fixed and the other one marches into the solution (see Figure 2.9).

Now we rework the solution to $x \sin(x) - 1 = 0$ using the method of false position and observe that it converges faster than the bisection method. Also, notice that $\{b_n - a_n\}_{n=0}^{\infty}$ does not go to zero.

Example 2.8. Use the false position method to find the root of $x \sin(x) - 1 = 0$ that is located in the interval $[0, 2]$ (the function $\sin(x)$ is evaluated in radians).

Starting with $a_0 = 0$ and $b_0 = 2$, we have $f(0) = -1.00000000$ and $f(2) = 0.81859485$, so a root lies in the interval $[0, 2]$. Using formula (22), we get

$$c_0 = 2 - \frac{0.81859485(2 - 0)}{0.81859485 - (-1)} = 1.09975017 \quad \text{and} \quad f(c_0) = -0.02001921.$$

The function changes sign on the interval $[c_0, b_0] = [1.09975017, 2]$, so we squeeze from the left and set $a_1 = c_0$ and $b_1 = b_0$. Formula (22) produces the next approximation:

$$c_1 = 2 - \frac{0.81859485(2 - 1.09975017)}{0.81859485 - (-0.02001921)} = 1.12124074$$

and

$$f(c_1) = 0.00983461.$$

Next $f(x)$ changes sign on $[a_1, c_1] = [1.09975017, 1.12124074]$, and the next decision is to squeeze from the right and set $a_2 = a_1$ and $b_2 = c_1$. A summary of the calculations is given in Table 2.2. ■

The termination criterion used in the bisection method is not useful for the false position method and may result in an infinite loop. The closeness of consecutive iterates and the size of $|f(c_n)|$ are both used in the termination criterion for Program 2.3. In Section 2.3 we discuss the reasons for this choice.

Table 2.2 False Position Method Solution of $x \sin(x) - 1 = 0$

k	Left endpoint, a_k	Midpoint, c_k	Right endpoint, b_k	Function value, $f(c_k)$
0	0.00000000	1.09975017	2.00000000	-0.02001921
1	1.09975017	1.12124074	2.00000000	0.00983461
2	1.09975017	1.11416120	1.12124074	0.00000563
3	1.09975017	1.11415714	1.11416120	0.00000000

Numerical Methods Using Matlab, 4th Edition, 2004
John H. Mathews and Kurtis K. Fink
ISBN: 0-13-065248-2

Prentice-Hall Inc.
Upper Saddle River, New Jersey, USA
<http://vig.prenhall.com/>

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FOURTH EDITION



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